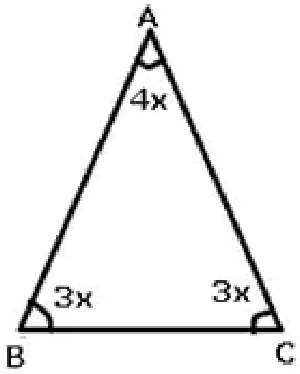
Chapter 12. Isosceles Triangle

Ex 12.1

Answer 1.



The equal angles and the non-equal angle are in the ratio 3:4.

Let equal angles be 3x each, therefore non-equal angle is 4x.

Angles of a triangle $= 180°$

$\Rightarrow 3x + 3x + 4x = 180°$

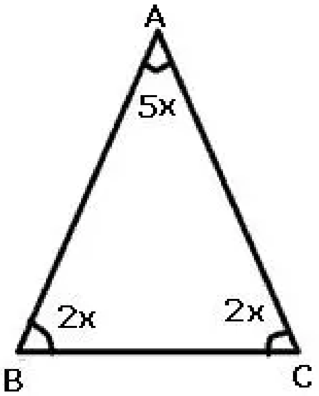
$\Rightarrow 10\ x = 180°$

$\Rightarrow x = 18°$

Therefore, $3x = 54°$ and $4x = 72°$

Angles $= 54°$, $54°$ and $72°$

Answer 2.



The equal angles and the non-equal angle are in the ratio 2:2:5.

Let equal angles be 2x each, therefore non-equal angle is 5x.

Angles of a triangle $=180^\circ$

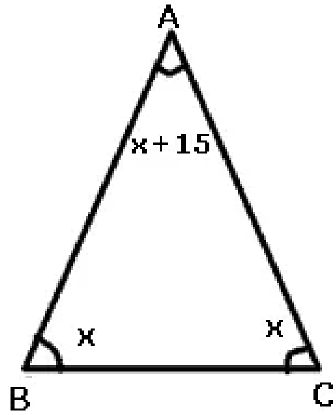
$\Rightarrow 2x + 2x + 5x = 180^\circ$
$\Rightarrow 9x = 180^\circ$

$\Rightarrow x = 20^\circ$

Therefore, $2x = 40^\circ$ and $5x = 100^\circ$

Angles $= 40^\circ, 40^\circ$ and 100°

Answer 3.



Let equal angles of the isosceles triangle be x each. Therefore, non-equal angle $= x+15^\circ$

Angles of a triangle $= 180^\circ$

$x + x + (x+15^\circ) = 180^\circ$

$3x + 15^\circ = 180^\circ$

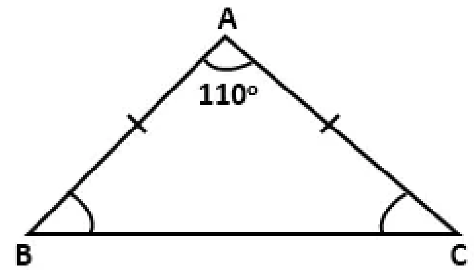
$3x = 165^\circ$

$x = 55^\circ$

$x+15 = 70^\circ$

Angles are $55^\circ, 55^\circ$ and 70°

Answer 4A.



In ABC,

$\angle A = 110°$

$AB = AC$

$\Rightarrow \angle C = \angle B$ ....(angles opposite to two equal sides are equal)

Now, by angle sum property,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + \angle B + \angle B = 180°$

$\Rightarrow 110° + 2\angle B = 180°$

$\Rightarrow 2\angle B = 180° - 110°$

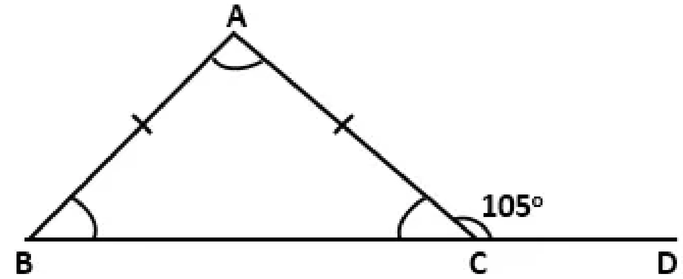
$\Rightarrow 2\angle B = 70°$

$\Rightarrow \angle B = 35°$

$\Rightarrow \angle C = 35°$

Hence, $\angle B = 35°$ and $\angle C = 35°$

Answer 4B.



In ABC,

$AB = AC$

$\Rightarrow \angle ACB = \angle ABC$ ....(1)(angles opposite to two equal sides are equal)

Now, $\angle ACB + \angle ACD = 180°$ ....(linear pair)

$\Rightarrow \angle ACB = 180° - \angle ACD$

$\Rightarrow \angle ACB = 180° - 105°$

$\Rightarrow \angle ACB = 75°$

$\Rightarrow \angle ABC = 75°$ ....[From (1)]

By angle sum property, in ΔABC

$\angle ABC + \angle ACB - \angle BAC = 180^\circ$

$\Rightarrow 75^\circ + 75^\circ + \angle BAC = 180^\circ$

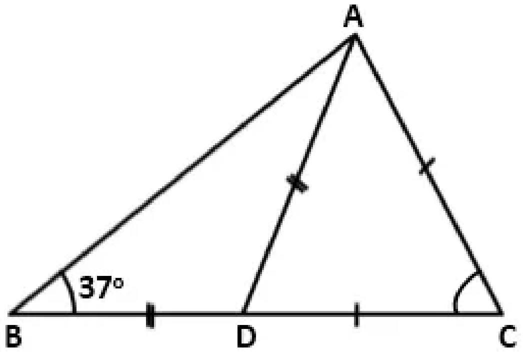
$\Rightarrow 150^\circ + \angle BAC = 180^\circ$

$\Rightarrow \angle BAC = 180^\circ - 150^\circ$

$\Rightarrow \angle BAC = 30^\circ$

Hence, in ΔABC, $\angle A = 30^\circ$, $\angle B = 75^\circ$ and $\angle C = 75^\circ$

Answer 4C.



In ΔABD,

$AD = BD$ ....(given)

$\Rightarrow \angle ABD = \angle BAD$ ....(angles opposite to two equal sides are equal)

Now, $\angle ABD = 37^\circ$ ....(given)

$\Rightarrow \angle BAD = 37^\circ$

By exterior angle property,

$\angle ADC = \angle ABD + \angle BAD$

$\Rightarrow \angle ADC = 37^\circ + 37^\circ = 74^\circ$

In ΔADC,

$AC = DC$ ....(given)

$\Rightarrow \angle ADC = \angle DAC$ ....(angles opposite to two equal sides are equal)

$\Rightarrow \angle DAC = 74^\circ$

Now, $\angle BAC = \angle BAD + \angle DAC$

$\Rightarrow \angle BAC = 37^\circ + 74^\circ = 111^\circ$

In ΔABC,

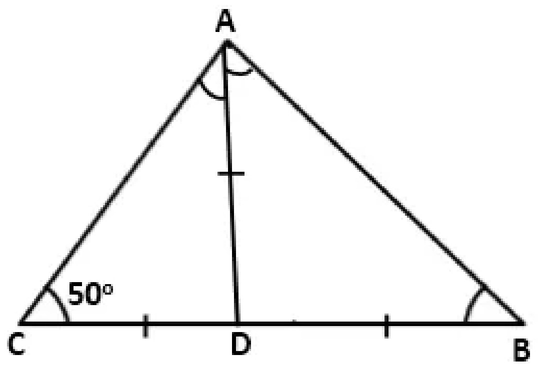
$\angle BAC + \angle ABC + \angle ACB = 180^\circ$

$\Rightarrow 111^\circ + 37^\circ + \angle ACB = 180^\circ$

$\Rightarrow \angle ACB = 180^\circ - 111^\circ - 37^\circ = 32^\circ$

Hence, the interior angles of ΔABC are 37°, 111° and 32°.

Answer 4D.



In ΔACD,

$AD = CD$ ....(given)

$\Rightarrow \angle ACD = \angle CAD$ ....(angles opposite to two equal sides are equal)

Now, $\angle ACD = 50°$ ....(given)

$\Rightarrow \angle CAD = 50°$

By exterior angle property,

$\angle ADB = \angle ACD + \angle CAD = 50° + 50° = 100°$

In ΔADB,

$AD = BD$ ....(given)

$\Rightarrow \angle DBA = \angle DAB$ ....(angles opposite to two equal sides are equal)

Also, $\angle ADB + \angle DBA + \angle DAB = 180°$

$\Rightarrow 100° + 2\angle DBA = 180°$

$\Rightarrow 2\angle DBA = 80°$

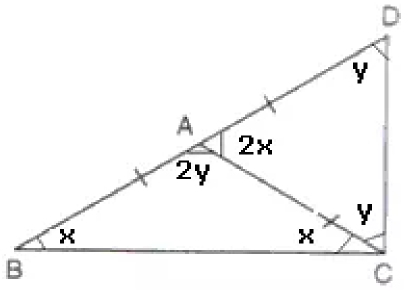
$\Rightarrow \angle DBA = 40°$

$\Rightarrow \angle DAB = 40°$

$\angle BAC = \angle DAB + \angle CAD = 40° + 50° = 90°$

Hence, the interior angles of ΔABC are $50°, 90°$ and $40°$.

Answer 5.



Let ∠ABC = x, therefore ∠BCA = x since AB = AC

In ΔABC,

∠ABC + ∠BCA + ∠BAC = 180° ......(i)

But ∠BAC + ∠DAC = 180° ......(ii)

From (i) and (ii)

∠ABC + ∠BCA + ∠BAC = ∠BAC + ∠DAC

∠DAC = ∠ABC + ∠BCA = x + x = 2x

Let ∠ADC = y, therefore ∠DCA = y since AD = AC

In ΔADC,

∠ADC + ∠DCA + ∠DAC = 180° ......(iii)

But ∠BAC + ∠DAC = 180° ......(iv)

From (iii) and (iv)

∠ADC + ∠DCA + ∠DAC = ∠BAC + ∠DAC

∠BAC = ∠ADC + ∠DCA = y + y = 2y

Substituting the value of ∠BAC and ∠DCA in (ii)

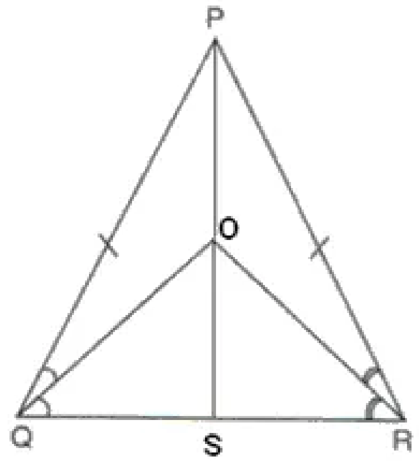
2x + 2y = 180°

x + y = 90°

⇒∠BCA + ∠DCA = 90°

⇒∠BCD is a right angle.

Answer 6.



Join PO and produce to meet QR in S.

In ΔPQS and ΔPRS

PS = PS (common)

PQ = PR (given)

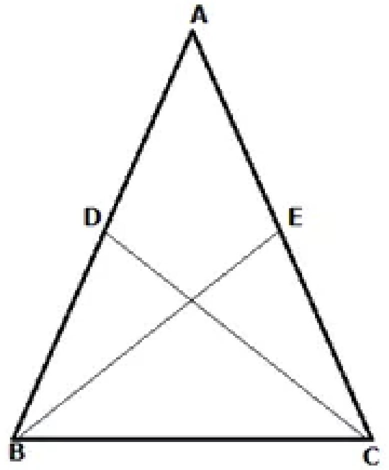
$\angle Q = \angle R$

$\therefore \Delta PQS \cong \Delta PRS$

$\therefore \angle QPS = \angle RPS$

Hence, PO bisects $\angle P$.

Answer 7.



Let ABC be an isosceles triangle with AB=AC.

Let D and E be the mid points of AB and AC.

Join BE and CD.

Then BE and CD are the medians of this isosceles triangle.

In ΔABE and ΔACD

$AB = AC$ (given)

$AD = AE$ (D and E are mid points of AB and AC)

$\angle A = \angle A$ (common angle)

Therefore, $\Delta ABE \cong \Delta ACD$ (SAS criteria)

Hence, $BE = CD$

Answer 8.

In ΔQDP,

$DP = DQ$

$\therefore \angle Q = \angle P$

$\angle QDR = \angle Q + \angle P$

$2\angle QDC = \angle Q + \angle P$ (DC bisects angle QDR)

$2\angle QDC = \angle Q + \angle Q = 2\angle Q$

$\angle QDC = \angle Q$

But these are alternate angles.

$\therefore DC \parallel PQ$

Answer 9.

In ΔPQS,

$PQ = PS$

$\therefore \angle PQS = \angle PSQ$

$\angle P + \angle PQS + \angle PSQ = 180°$

$50° + 2\angle PQS = 180°$

$2\angle PQS = 130°$

$\angle PQS = 65° = \angle PSQ$ ....(i)

In ΔSRQ,

$SR = RQ$

$\therefore \angle RQS = \angle RSQ$

$\angle R + \angle RQS + \angle RSQ = 180°$

$110° + 2\angle RQS = 180°$

$2\angle RQS = 70°$

$\angle RQS = 35^\circ = \angle RSQ$ ....(ii)

Adding (i) and (ii)

$\angle PSQ + \angle RSQ = 65^\circ + 35^\circ$

$\angle PSR = 100^\circ$

Answer 10.

In $\triangle BDC$,

$\angle BDC = 70^\circ$

$BD = BC$

Therefore, $\angle BDC = \angle BCD$

$\Rightarrow \angle BCD = 70^\circ$

Now $\angle BCD + \angle BDC + \angle DBC = 180^\circ$

$70^\circ + 70^\circ + \angle DBC = 180^\circ$

$\angle DBC = 40^\circ$

$\angle DBC = \angle ABC$ (BC is the angle bisector)

$\Rightarrow \angle ABC = 40^\circ$

In $\triangle ABC$,

Since $AB = AC$, $\angle ABC = \angle ACB \Rightarrow \angle ACB = 40^\circ$

$\angle ACB + \angle ABC + \angle BAC = 180^\circ$

$40^\circ + 40^\circ + \angle BAC = 180^\circ$

$\angle BAC = 100^\circ$

Hence, $\angle BAC = 100^\circ$ and $\angle DBC = 40^\circ$

Answer 11.

(i) In ΔPQR,

$PQ = PR$ (given)

$\therefore \angle R = \angle Q$ ....(i)

Now in ΔQNT and ΔRMT

$\angle QNT = \angle RMT$ (90°)

$\angle Q = \angle R$ (from (i))

$QT = TR$ (given)

$\therefore \Delta QNT \cong \Delta RMT$ (AAS criteria)

$\therefore TN = TM$

(ii) Since, $\Delta QNT \cong \Delta RMT$

$NQ = MR$ ....(ii)

But $PQ = PR$ ....(iii) (given)

Subtracting (ii) from (iii)

$PQ - NQ = PR - MR$

$\Rightarrow PN = PM$

(iii) In ΔPNT and ΔPMT

$TN = TM$ (proved)

$PT = PT$ (common)

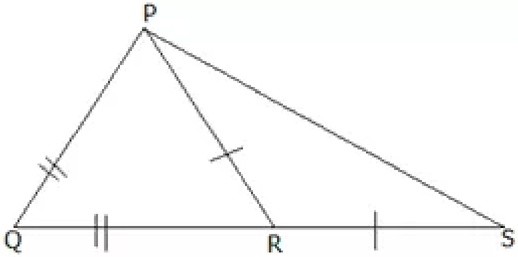
$\angle PNT = \angle PMT$ (90°)

Therefore, $\Delta PNT \cong \Delta PMT$

Hence, $\angle NPT = \angle MPT$

Thus, PT bisects $\angle P$

Answer 12.



In ΔPQR,

$PQ = QR$ (given)

$\angle PRQ = \angle QPR$ ......(i)

In ΔPRS,

$PR = RS$ (given)

$\angle PSR = \angle RPS$ ......(ii)

Adding (i) and (ii)

$\angle QPR + \angle RPS = \angle PRQ + \angle PSR$

$\angle QPS = \angle PRQ + \angle PSR$ ....(iii)

Now in ΔPRS,

$\angle PRQ = \angle RPS + \angle PSR$

$\angle PRQ = \angle PSR + \angle PSR$ (from (ii))

$\angle PRQ = 2\angle PSR$ ....(iv)

Now, $\angle QPS = 2\angle PSR + \angle PSR$ (from (iii) and (iv))

$\angle QPS = 3\angle PSR$

$$\frac{\angle PSR}{\angle QPS} = \frac{1}{3}$$

$\Rightarrow \angle PSR : \angle QPS = 1 : 3$

Answer 13.

In ΔKTM,

$KT = TM$ (given)

Therefore, $\angle TKM = \angle TMK$ .....(i)

Now, $\angle KTL = \angle TKM + \angle TMK$

$80° = \angle TKM + \angle TKM = 2\angle TKM$ (from (i))

$\angle TKM = 40° = \angle TMK = \angle LMK$ ....(ii)

But $\angle TKM = \angle TKL$ (KT is the angle bisector)

Therefore, $\angle TKL = 40°$

In ΔKTL,

$\angle TKL + \angle KTL + \angle KLT = 180°$

$40° + 80° + \angle KLT = 180°$

$\angle KLT = 60° = \angle KLM$

$\angle KLM = 60°$ and $\angle LMK = 40°$

Answer 14

In ΔPTQ and ΔPSR

$PQ = PR$ (given)

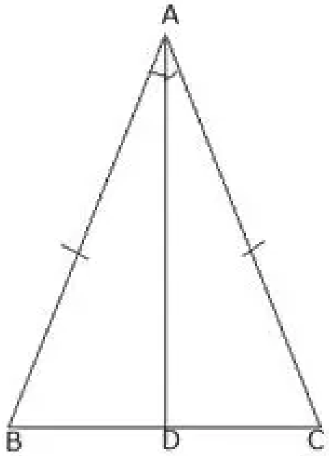
$PT = PS$ (given)

$\angle TPQ = \angle SPR$ (vertically opposite angles)

Therefore, $\Delta PTQ \cong \Delta PSR$

Hence, $TQ = SR$

Answer 15.



In $\triangle ADB$ and $\triangle ADC$

$AB = AC$ (given)

$AD = AD$ (common)

$\angle BAD = \angle CAD$ (AD bisects $\angle BAC$)

Therefore, $\triangle ADB \cong \triangle ADC$

Hence, $BD = DC$ and $\angle BDA = \angle CDA$

But $\angle BDA + \angle CDA = 180^\circ$

$\Rightarrow \angle BDA = \angle CDA = 90^\circ$

Therefore, AD bisects BC perpendicularly.

Answer 16.

In ΔDEC,

$\angle DEC = \angle ADE + \angle A = 2a$ (ext. Angle to ΔADE)

$DE = DC$

$\Rightarrow \angle DEC = \angle DCE = 2a$ ....(ii)

In ΔBDC, let $\angle B = b$

$DC = BC$

$\Rightarrow \angle BDC = \angle B = b$ ....(iii)

In ΔABC,

$\angle ADB = \angle ADE + \angle EDC + \angle BDC$

$180^\circ = a + \angle EDC + b$ (from (i) and (iii))

$\angle EDC = 180^\circ - a - b$ ......(iv)

Now again in ΔDEC

$180^\circ = \angle EDC + \angle DCE + \angle DEC$ (from (ii))

$180^\circ = \angle EDC + 2a + 2a$

$\angle EDC = 180^\circ - 4a$ ....(v)

Equating (iv) and (v)

Answer 17.

(i) In ΔADC,

$AD = AC$ (given)

Therefore, $\angle ADC = \angle ACD$ ....(i)

But $\angle ADB + \angle ADC = 180^\circ$ ....(ii)

And $\angle ACD + \angle DCE = 180^\circ$ ....(iii)

From (i), (ii) and (iii)

$\angle ADB = \angle DCE$

(ii) In ΔABD and ΔDCE

$BD = CD$ (D is mid-point of BC)

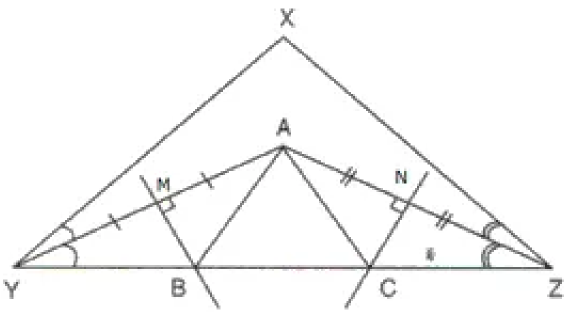
$\angle ADB = \angle DCE$ (proved)

$AD = CE$ (since $AD = AC$ and $AC = CE$)

Therefore, $\Delta ABD \cong \Delta DCE$

Hence, $AB = CE$.

Answer 18.



Let M and N be the points where AY and AZ are bisected.

In ΔABM and ΔBMY

MY = MA (BM bisects AY)

BM = BM (common)

$\angle BMY = \angle BMA$

Therefore, $\Delta ABM \cong \Delta BMY$

Hence, YB = AB ....(i)

In ΔACN and ΔCNZ

NZ = NA (CN bisects AZ)

CN = CN (common)

$\angle CAN = \angle CNZ$

Therefore, $\Delta ACN \cong \Delta CNZ$

Hence, CZ = AC ....(ii)

YZ = YB + BC + CZ

Substituting from (i) and (ii)

YZ = AB + BC + AC

Hence, YZ is equal to the perimeter of ΔABC

Answer 19.

In ΔPQR, let $\angle PQR = x$

$PQ = PR$

$\Rightarrow \angle PQR = \angle PRQ = x$ ....(i)

In ΔRNS,

$\angle NRS = \angle PRQ = x$ ....(vertically opposite angles)

$\angle RNS = 90°$ (given)

$\angle NSR + \angle RNS + \angle NRS = 180°$

$\angle NSR + 90° + x = 180°$

$\angle NSR = 90° - x$ ....(ii)

Now in Quadrilateral PTRS

$\angle PTS = 90°$ (given)

$\angle TPR = \angle PQR + \angle PRQ = 2x$ (exterior angle to triangle PQR)

$\angle PRS = 180° - \angle PRQ = 180° - x$ (QRS is a st. Line)

$\angle PTS + \angle TPR + \angle PRS + \angle TSR = 360°$ (angles of a quad. = 360°)

$90° + 2x + 180° - x + \angle TSR = 360°$

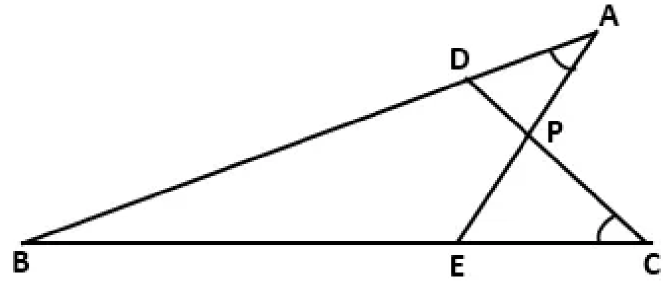
$\angle TSR = 90° - x$ ....(iii)

From (ii) and (iii)

$\angle TSR = \angle NSR$

Therefore, QS bisects $\angle TSN$.

Answer 20.



Join DE and AC.

In ΔAPD and ΔEPC,

$\angle DAP = \angle ECP$ ....($\because \angle BAE = \angle BCD$)

$AP = CP$ ....(given)

$\angle APD = \angle EPC$ ....(vertically opposite angles)

$\therefore \Delta APD \cong \Delta EPC$ ....(By ASA Congruence criterion)

$\Rightarrow AD = EC$ ....(c.p.c.t)

In ΔAPC,

$AP = CP$ ....(given)

$\Rightarrow \angle PAC = \angle PCA$ ....(angles opposite to two equal sides are equal)

Now, $\angle BAE = \angle BCD$ and $\angle PAC = \angle PCA$

$\Rightarrow \angle BAC = \angle BCA$

$\Rightarrow BC = BA$ ....(sides opposite to two equal angles are equal)

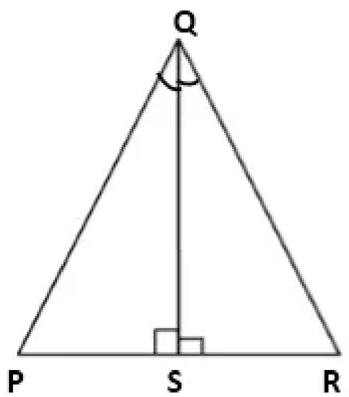
$\Rightarrow BE + EC = BD + DA$

$\Rightarrow BE = BD$ ....($\because EC = DA$)

$\Rightarrow \angle BDE = \angle BED$ ....(angles opposite to two equal sides are equal)

$\Rightarrow \Delta BDE$ is an isosceles triangle.

Answer 21.



In ΔPQS and ΔSQR,

$QS = QS$ ....[Common]

$\angle QSP = \angle QSR$ ....[each = $90°$]

$\angle PQS = \angle RQS$ ....[given]

$\therefore \Delta PQS \cong \Delta SQR$ ....[By ASA criterion]

$\Rightarrow PS = RS$

$\Rightarrow x + 1 = y + 2$

$\Rightarrow x = y + 1$ ....(i)

And PQ = SQ
$\Rightarrow 3x + 1 = 5y - 2$
$\Rightarrow 3(y + 1) + 1 = 5y - 2$ ....[From (i)]
$\Rightarrow 3y + 3 + 1 = 5y - 2$
$\Rightarrow 3y + 4 = 5y - 2$
$\Rightarrow 2y = 6$
$\Rightarrow y = 3$
Putting y = 3 in (i),
$x = y + 1 = 3 + 1 = 4$

Answer 22A.

a.
In ΔPQS,
PQ = QS ....(given)
$\Rightarrow \angle QSP = \angle QPS$ ....(angles opposite to two equal sides are equal)
Now, $\angle PQS + QSP + \angle QPS = 180°$
$\Rightarrow 60° + 2\angle QSP = 180°$
$\Rightarrow 2\angle QSP = 120°$
$\Rightarrow \angle QSP = 60°$
$\Rightarrow \angle QPS = 60°$

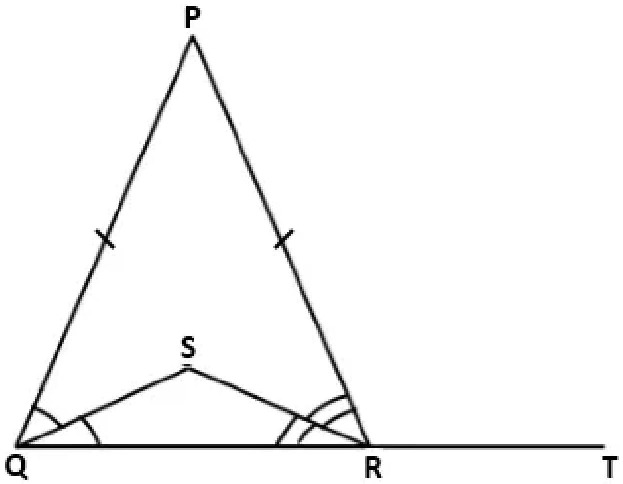
In ΔPRS,
PS = SR ....(given)
$\Rightarrow \angle PRS = \angle RPS$ ....(angles opposite to two equal sides are equal)

By exterior angle property,
$\angle QSP = \angle RPS + \angle PRS$
$\Rightarrow 60° = 2\angle RPS$
$\Rightarrow \angle RPS = 30°$

Now, $\angle QPR = \angle QPS + \angle RPS = 60° + 30° = 90°$

b.
In ΔPQS,
$\angle PQS = 60°$, $\angle QPS = 60°$ and $\angle QSP = 60°$
$\Rightarrow$ ΔPQS is an equilateral triangle.
$\Rightarrow PQ = QS = PS$
And, PS = SR
$\Rightarrow PQ = PS = QS = SR$

Answer 23.



Let $\angle PQS = \angle SQR = x$ and $\angle PRS = \angle SRQ = y$

In ΔPQR,

$\angle QPR + \angle PQR + \angle PRQ = 180°$

$\Rightarrow \angle QPR + 2x + 2y = 180°$

$\Rightarrow \angle QPR = 180° - 2x - 2y$ ....(i)

Since PQ = PR,

$\angle PRQ = \angle PQR$ ....(angles opposite to two equal sides are equal)

$\Rightarrow 2x = 2y$

$\Rightarrow x = y$

Now, $\angle PRT = \angle PQR + \angle QPR$ ....(by exterior angle property)

$\Rightarrow \angle PRT = 2x + 180° - 2x - 2y$ ....[From (i)]

$\Rightarrow \angle PRT = 180° - 2y$ ....(ii)

In ΔSQR,

$\angle QSR + \angle SQR + \angle SRQ = 180°$

$\Rightarrow \angle QSR + x + y = 180°$

$\Rightarrow \angle QSR = 180° - x - y$

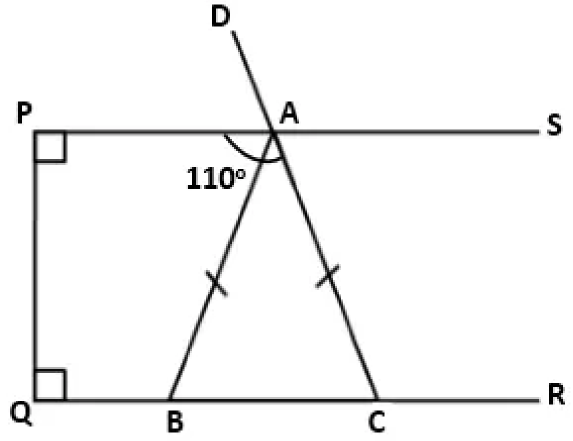
$\Rightarrow \angle QSR = 180° - y - y$ ....[$\because x = y$ (proved)]

$\Rightarrow \angle QSR = 180° - 2y$ ....(iii)

From (ii) and (iii),

$\angle QSR = \angle PRT$

Answer 24.



Given : $\angle PAC = 110°$

To find:

Base angles: $\angle ABC$ and $\angle ACB$

Vertex angle: $\angle BAC$

In quadrilateral APQC,

$\angle APQ + \angle PQC + \angle ACQ + \angle PAC = 360°$

$\Rightarrow 90° + 90° + \angle QCA + 110° = 360°$

$\Rightarrow \angle ACQ = 360° - 290°$

$\Rightarrow \angle ACQ = 70°$

$\Rightarrow \angle ACB = 70°$ ....(i)

In ΔABC,

$AB = AC$ ....(given)

$\Rightarrow \angle ACB = \angle ABC$ ....(angles opposite to two equal sides are equal)

$\Rightarrow \angle ABC = 70°$ ....[From (i)]

In ΔABC,

$\angle ABC + \angle ACB + \angle BAC = 180°$ ....(angle sum property)

$\Rightarrow 70° + 70° + \angle BAC = 180°$

$\Rightarrow \angle BAC = 180° - 140°$

$\Rightarrow \angle BAC = 40°$